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Time analogue of the z-scan technique suitable to waveguides

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Abstract. We propose a new technique to perform measurements of nonlinearities in optical waveguides based on the variation of an optical pulse chirp. This technique is analogous to the z-scan technique in the time domain. We analyze the new experimental method analytically and numerically, obtaining an useful expression relating the nonlinearity with a peak-valley structure. Practical ways to implement the technique are discussed.

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The z-scan technique is a well established technique to characterize optical nonlinearities of bulk materials [1]. Many variants have been developed for different kinds of media [2], including absorbing ones [3], and using different beam profiles as top hats [4], etc. Nevertheless these methods are not useful for measurements of nonlinearities in optical waveguides. In this article we propose a novel technique which is an extension of the z-scan to the time domain, opening the possibility to use this powerful method in waveguides. Here we will use a fundamental analogy observed very early in the development of nonlinear optics [5], the equivalence between space and time in the equations describing the propagation of pulses in waveguides and the propagation of a profile in the space using the paraxial approximation. When there is only one transverse spatial dimension, the analogy is straightforward. In the nonlinear waveguide environment, for example, spatial solitons [6,7], modulation instability [8,9] and beam profile compression [9–11] have all been reported. For two transverse dimensions, the Kerr effect combined with diffraction leads to self-focusing or self-defocusing of the beam depending on the sign of n_2. If the bulk medium is thin as in the z-scan, only a change in the phase of the beam profile is caused, but this effect revealed itself very useful in the determination of the sign and magnitude of the nonlinearity.

In the z-scan technique we make a measurement of the intensity passing through one aperture (localized at a far field distance) of a focalized Gaussian beam, when we move a sample in the z-direction. If we consider the effects of a converging thin lens over a collimated Gaussian beam we can show that a quadratic phase dependence is imposed over the beam profile [12], with an opposite sign of the quadratic phase generated by diffraction in the propagation in free space. This quadratic phase will cause a focalization of the beam until a minimum spot, followed by an expansion with a quadratic phase growing up again since diffraction keeps doing its job. The nonlinear sample, inserted in the beam path, produces a nonlinear change in wave front which appears at the far field, but does not change the radial or angular amplitude distribution at the sample.

The focalization of a Gaussian beam is completely analogous to the compression of a Gaussian pulse with quadratic phase dependence (usually described as a linear frequency scanning or chirp) opposite to the group velocity dispersion [13,14]. In our method [15] the sample scanning in the z-direction of the z-scan method is substituted by a variation in the linear chirp of an optical pulse, and the transmission measurement by an aperture is accomplished by the energy integration over a spectral window.

In order to formally establish the analogy we suppose an initial chirped Gaussian pulse:

$$A(0,t) = \sqrt{I_0}\exp\left[-\frac{1+\mathrm{i}C_0}{2}\frac{t^2}{T_0^2}\right] \tag{1}$$

where I_0 is the initial pulse peak intensity, T_0 is the pulse duration and C_0 is a positive defined constant associated to the phase and related to the frequency scanning through $\delta\omega = -\partial\phi/\partial t$. Obviously as C_0 is a positive constant, the pulse has a quadratic phase dependence with time similar to the one imposed by the lens in the Gaussian beam of the z-scan. Let us consider this pulse propagating in a dispersive fiber waveguide with anomalous dispersion. At the output of a fiber with a longitudinal dimension of L we have:

$$A(L,t) = \sqrt{\frac{I_0(1-\mathrm{i}C_0)}{1-\mathrm{i}C_{\mathrm{D}}}}\exp\left[-\frac{1+\mathrm{i}C_{\mathrm{D}}}{2}\frac{t^2}{T_1^2}\right] \tag{2}$$

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where:

$$C_{\mathrm{D}} = C_0 - \mathrm{sgn}\,(\beta_2)\,(1 + C_0^2)\frac{L}{L_{\mathrm{D}}} \quad (3)$$

$$T_1 = T_0\left[\left(1 + \frac{C_0\beta_2 L}{T_0^2}\right)^2 + \left(\frac{\beta_2 L}{T_0^2}\right)^2\right]^{\frac{1}{2}} \quad (4)$$

where β_2 is the group velocity dispersion and $L_{\mathrm{D}} = T_0^2/|\beta_2|$ is the dispersion length [13]. Now we have our focalized Gaussian beam in the time domain, but we still need to put the nonlinear sample waveguide "inside the beam" and travel with it in the z-direction. To do so, we get the optical pulse with a linear chirp coming out from the dispersive waveguide with "adjustable" length L and couple that pulse into the nonlinear optical waveguide under investigation. Mathematically it means that one is solving the problem of propagation of the pulse, given by the equation (2), in a nonlinear waveguide in the limit where the dispersive effects are negligible, obtaining:

$$A(\ell_n, t) = A(L,t)\exp\left[\mathrm{i}\gamma\,|A(L,t)|^2\,\ell_n\right]$$
$$\simeq \sqrt{\frac{I_0\exp\left[\mathrm{i}2\Phi_{\mathrm{NL}}^0\right](1 - \mathrm{i}C_0)}{1 - \mathrm{i}C_{\mathrm{D}}}}\exp\left[-\frac{1 + \mathrm{i}C_{\mathrm{NL}}}{2}\frac{t^2}{T_1^2}\right] \quad (5)$$

where $\Phi_{\mathrm{NL}}^0 = \gamma I_0 \ell_n T_0/T_1$ is the nonlinear phase at the pulse center, $C_{\mathrm{NL}} = C_{\mathrm{D}} + 2\Phi_{\mathrm{NL}}^0$, ℓ_n is the nonlinear waveguide length and γ is related to the nonlinear-index coefficient in a standard way [13] and in principle can be different of the usual Kerr coefficient. As the self-phase modulation effects are dominant, we may see that the nonlinearity affects only the phase (or chirp), without any modification of the pulse shape. This situation is completely analogous to the situation of the beam profile just after it crosses the sample in the z-scan technique. In order to get the aperture transmission in the time domain, for each "z-position" of the sample, we must obtain the equivalent of the far field, which is simply the Fourier transform of the pulse coming out of sample waveguide.

We define our equivalent to z-direction in the time as $\eta = L/L_{\mathrm{D}}$ and the nonlinear phase imposed by the sample as $\Delta = \gamma P_0 \ell_n$ where P_0 is the pulse peak power. The transmission through the time domain aperture will be taken as the transmission at the central peak of the spectrum (an infinitesimal aperture), which normalizing to the central peak of the spectrum before the sample, produces:

$$\Im(\eta) = \sqrt{1 + C_0^2}\left\{\left[1 + \frac{2\eta\Delta}{\left[(1 + \eta C_0)^2 + \eta^2\right]^{\frac{3}{2}}}\right]^2 + \left[C_0 + \frac{2(1 + \eta C_0)\,\Delta}{\left[(1 + \eta C_0)^2 + \eta^2\right]^{\frac{3}{2}}}\right]^2\right\}^{-\frac{1}{2}}. \quad (6)$$

In Figure 1 we have depicted the transmission at the central peak of the wavelength output spectrum when we varied the dispersive factor η, showing a similar variation as observed in the standard z-scan technique. In the case showed in the figure, the total nonlinear phase shift was only 200 milliradians, produced by the propagation in a sample waveguide with a positive nonlinearity. The central peak to valley transmission difference and nonlinearity relation, which is the main result of this article, is obtained from the previous equation:

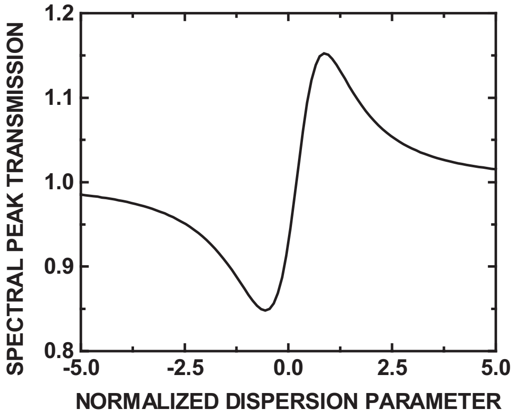

Fig. 1. Spectral central peak window transmission against normalized dispersion parameter.

$$\Delta T_{\mathrm{P-V}} = \frac{8\sqrt{3}}{9}\Delta. \quad (7)$$

It should be noted that in time domain a controllable quadratic phase variation (a linear chirp) must be generated in a different way that in the space domain. In fact the problem is how to obtain in practice the change in the signal chirp, equivalent to the situation in the z-scan when the sample crosses the beam waist point. A practical way to overcome this problem can be devised using a fiber-grating pair compressor. Let us consider an intense optical pulse propagating through an optical fiber, generating new frequencies through SPM with dispersive effects acting together, followed by a pair of diffraction gratings as in a fiber-grating compressor. The grating pair alone is not enough to generate the desired chirps, because there is no configuration capable to produce both positive and negative chirps in a continuous way. However when the grating pair distance is perfectly adjusted in a fiber-grating compressor to obtain the shortest pulse coming out, we have a resulting pulse chirpless [16].

In order to quantify this problem, we used the beam propagation method to obtain the pulse coming out of the fiber-grating compressor which will be coupled into the nonlinear sample waveguide. The points in Figure 2 represent the normalized spectral peak transmission obtained by propagating the output of the fiber-grating compressor (with a 2.5 radians nonlinear phase shift imposed by SPM in the optical fiber) through the sample waveguide for different grating separations. We observe a different shape from the one obtained in Figure 1. However, the peak to valley transmission variation still gives us the nonlinearity value of the sample waveguide, agreeing with the equation (7).

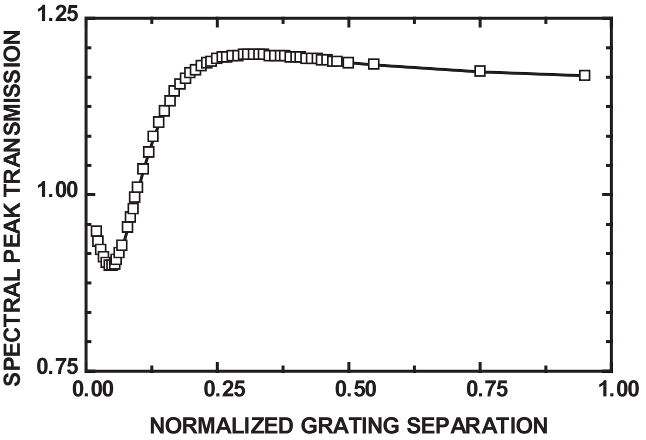

Fig. 2. Spectral central peak transmission as a function of the normalized grating separation.

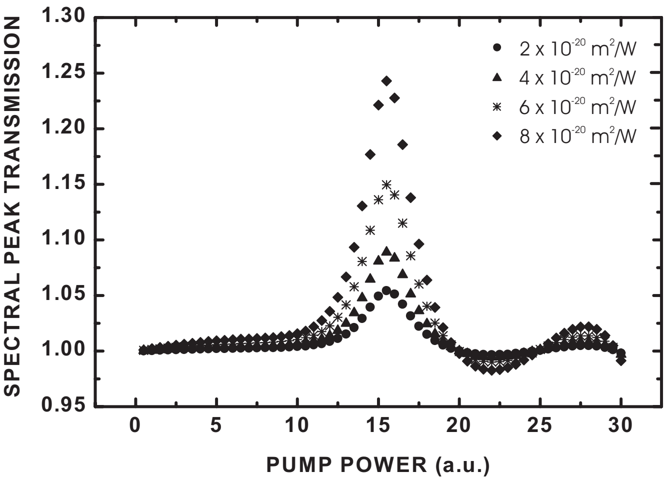

Fig. 3. Spectral central peak window transmission against pump power to different nonlinear index.

Another experimental alternative is using the same experimental setup and instead of moving the gratings, change the input power of the pulse entering the fiber-grating compressor. Initially adjusting conveniently the grating pair to obtain a perfectly compressed pulse without using the maximum input power available, and then increasing and decreasing the fiber-grating compressor input power, we may also obtain the two different regimes of chirp coming out of the grating pair. In fact now we have the time analogue of a nonlinear lens and we dynamically change the focal distance controlling the input power. In Figure 3 we show a few transmission curves calculated for different amounts of nonlinearity and clearly the same peak-valley feature is present. It is not obvious that is still possible to obtain the nonlinearity from the peak to valley transmission variations of these curves since we are changing the coupled power into the nonlinear waveguide and the resulting nonlinear phase shift is variable too. However we should remember that usually a nonlinear refractive index is independent of the intensity and in fact can be determined by the ratio between the

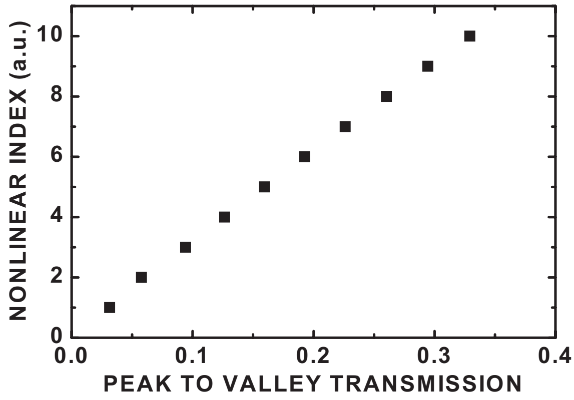

Fig. 4. Nonlinear index against peak to valley transmission variation to different nonlinearity values.

nonlinear phase and input power necessary to originate the phase, *i.e.* $\gamma = \Delta/P_0\ell_n$. To prove that, in Figure 4 we depicted the numerically obtained relation between the peak-valley transmission and the sample nonlinearity, showing a linear relationship between them and reassuring that only the measure of the peak to valley transmission variation is enough to obtain the value of the nonlinear index. These experimental setups can in principle be made using 10 ps duration pulses with peak powers of a few Watts in near infrared spectral region and standard grating with separations of a few ten's of centimeters.

In conclusion, we have proposed a novel method to determine the waveguide optical nonlinearities using an optical pulse with a variable chirp. We obtained an analytical expression relating the peak to valley spectral window transmission variation with the optical nonlinearity. Numerical results obtained using realistic experimental setups based on a fiber-grating compressor with a variable grating separation or variable input pulse power, agreed quite well with the analytical expression, showing that this method can be implemented to practical applications.

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